

The volume of a 4-sided  
~~pyramid~~ pyramid with a square base  
 $V = \frac{1}{3} a^2 h$

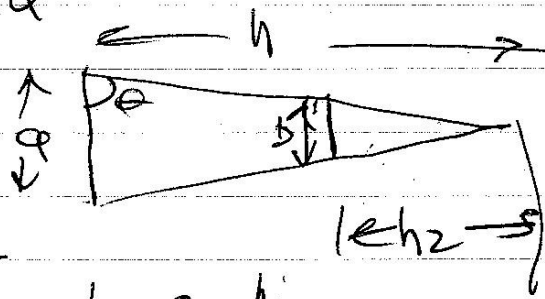
For our problem we want to find  $h_1$   
 when  $a = \frac{5}{8}$ ;  $b = \frac{3}{8}$  and the volume of  
 the original bar is  $7 \times \frac{5}{8} \times \frac{5}{8}$  or  $2a^2$

The volume of the truncated pyramid is  
 the volume of the whole pyramid - volume of the  
 top pyramid.

$$V = \frac{1}{3} a^2 h - \frac{1}{3} b^2 h_2 \quad \text{and} \quad h = h_1 + h_2$$

$$\text{Eq 1 so: } V = \frac{1}{3} a^2 (h_1 + h_2) - \frac{1}{3} b^2 h_2$$

Now Eye look at one side of the pyramid



$$\tan \theta = \frac{h}{\frac{a}{2}} \quad \tan \theta = \frac{h_2}{\frac{b}{2}}$$

So

$$\frac{h}{\frac{a}{2}} = \frac{h_2}{\frac{b}{2}}$$

or

$$\frac{h}{a} = \frac{h_2}{b} \quad \text{and} \quad h = h_1 + h_2$$

~~$\frac{h}{a} = \frac{h_2}{b}$~~

$$\frac{h_1 + h_2}{a} = \frac{h_2}{b}$$

$$bh_1 + bh_2 = ah_2$$

$$bh_1 = h_2(a - b)$$

$$\text{Eq II} \quad h_2 = \frac{b}{(a-b)} h_1$$

Repeating Eq I

$$V = \frac{1}{3} a^2 (h_1 + h_2) - \frac{1}{3} b^2 h_2$$

Replace  $h_2$  using Eq. II

$$V = \frac{a^2}{3} \left(1 + \frac{b}{a-b}\right) h_1 - \frac{b^2}{3} h_1$$

$$= \frac{b^2}{3} \left(\frac{b}{a-b}\right) h_1$$

$$V = h_1 \left[ \frac{a^2}{3} \left(1 + \frac{b}{a-b}\right) - \frac{b^2}{3(a-b)} \right]$$

$$h_1 = \frac{3V}{a^2 \left(1 + \frac{b}{a-b}\right) - \frac{b^2}{a-b}} \quad \text{and} \quad V = L a^2$$

Plug in the Dimensions

$$h_1 = 10.71''$$

$$h_c = \frac{3La^2}{a^2 \left(1 + \frac{b}{a-b}\right) - \frac{b^3}{a-b}}$$

$$h_c = \frac{3L}{\left(1 + \frac{b}{a-b}\right) - \frac{b^3}{a^2(a-b)}}$$

$$\begin{aligned} L &= 7 \\ a &= 5/8 \\ b &= 3/8 \end{aligned}$$

$$h_c = \frac{21}{\left(1 + \frac{3/8}{5/8 - 3/8}\right) - \frac{(3/8)^3}{(5/8)^2(5/8 - 3/8)}}$$

$$h_c = \frac{3L}{\left(1 + \frac{3}{2}\right) - \frac{3^3}{5^2(2)}} = \frac{3L}{2.5 - \frac{27}{50}}$$

$$h_c = \frac{21}{1.96} = 10.71$$